Stern, J. (2019). Miloš Adžić and Kosta Došen, eds, *Gödel's Basic Logic Course at Notre Dame*, Belgrade: Logical Society Belgrade, Dosije, 2017, 302 pp., ISBN 978-8660472399. *Dialectica*, *72*(4), 617-622. https://doi.org/10.1111/1746-8361.12247

Logic Lectures. Gödel's Basic Logic Course at Notre Dame edited by Miloš Adžić and Kosta Došen

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This book is at treat. The logic community should thank Miloš Adžić and the late Kosta Došen for their huge effort in editing Gödel's lecture notes of his 1939 Basic Logic Course at Notre Dame. Adžić and Došen have turned a number of handwritten notebooks that served as the basis of Gödel's course into—even by modern standards—an elegant logic textbook, which is a pleasure to read. The edited lecture notes highlight once again what a clear and lucid thinker Gödel was.

The book consists of an editorial introduction in which the editorial decisions and the notational conventions are explained, the edited text of Gödel's logic course, and the source text in printed form. The source text provides a good idea of the amount of work that has gone into the editing process. Gödel's notes are full of abbreviations, a number of passages are crossed out and every now and then there are interludes in which Gödel is concerned with seemingly unrelated issues—for example religious questions. The material covered by Gödel consists of a thorough discussion of propositional logic, an introduction to first-order logic and to basic notions of the calculus of classes. The lecture notes end with a short discussion of Russell's paradox and the theory of types. In this review I shall start by briefly commenting on the editorial introduction and a number of Adžić and Došen's editorial decisions. I then provide a brief summary and discussion of the content of Gödel's Basic Logic Course. Before I start I would like to point the interested reader to the work of Adžić and Došen (2016) and Cassou-Nogues (2009). These works provide an outline of the content of the Gödel's Notre Dame lectures and comment on their philosophical and historical significance.

Editorial Work and Decisions In the editorial introduction Adžić and Došen clearly lay out their editorial decisions and notational practices. This concerns, for example, the structure and order of the content in the edited text, which requires some rearrangement in comparison to the source text, that is, the handwritten notes. Some of the reshuffling is suggested by various page numbering systems used by Gödel, others seems to be required by content and formulations in the text. In general, Adžić and Došen employ a very conservative editing policy in that they stay as close as possible to the source text. At places, in particular the beginning of the lecture (pp. 1-23) and the discussion of the paradoxes (pp. 106-115) this leads to the reproduction of two versions of the same content within the edited version. In my opinion this is slightly unfortunate since I doubt Gödel covered the same material twice during his lecture. More importantly, including only one version in the main text and relegating the other version, perhaps in form of an appendix, to the end of the edited text would have made for an even more enjoyable read.

In contrast to their otherwise very conservative editorial policy the editors have sometimes, as they clearly indicate, included passages in the edited version of Gödel's lecture notes that have been crossed out in the handwritten text. I am not entirely convinced that this more intrusive stance is warranted. While it seems fine to include such passages by means of footnotes, as for example in Footnote 7, integrating them into the main body of the text seems to be more problematic, as they were not intended to be part of the lecture. One example is the discussion of definite descriptions and Russell's infamous 'The King of France is bald'-example (pp. 106-107), which could have been relegated to a footnote. However, these are a stickler's complaints and should not distract from Adžić and Došen's sound editorial work, which make the edited text an enjoyable read.

In the editorial introduction Adžić and Došen confine themselves to explaining their editorial decisions and practices, and only give a very brief outline of the lecture's content. This contrasts, for example, with the presentation of Gödel's papers in his *Collected Works*, edited by Feferman et al., where each contribution is accompanied by an editorial putting Gödel's work into its scientific and historical context. Personally, I always find it interesting to gain perspective and to learn more about the scientific-historical context. The fact that this aspect is missing almost entirely from the editorial introduction is then my only slight disappointment with respect to this book. Adžić and Došen justify their omission by pointing towards their also very brief discussion in Adžić and Došen (2016). But why not reproduce or integrate the paper in the editorial introduction of the book?

Content and Discussion Gödel's logic course is from a contemporary perspective surprisingly standard and could, after some modifications, still be used as an introductory logic text. It roughly follows the outlines of Hilbert and Ackermann (1928) but supersedes their presentation with respect to clarity and accessibility. As mentioned, the lecture covers soundness and completeness of propositional logic, introduces first-order logic and some basic notions of the theory of classes.

Gödel starts the lecture by characterizing logic as the *"science of the laws of correct thinking"* and from this characterization infers *"the theory of inference"* and *"the theory of logically true propositions"* as the central parts of logic. The target Gödel sets himself for the lectures is to *"fill two gaps of traditional logic, i.e., 1. to provide as far as possible a complete theory of logical inference and of logically true propositions and 2. to show how all of them can be deduced from a minimum number of primitive laws."*[1]

Throughout his lecture Gödel contrasts state-of-the art logic with traditional approaches—in particular Aristotelean syllogistic but, arguably, also Whitehead and Russell's *Principia*, where the two gaps we have just mentioned are not addressed—and spends some time and effort on showing how modern logic supersedes the traditional approaches. A prominent example is the discussion of the existential presuppositions of a number of Aristotelean moods,

[1] Compare pp. 1-2 and, especially, pp. 8-9.

which are not valid modes of inference in the sense of first-order predicate logic because they preclude the possibility of predicates with an empty extension. A plausible explanation for this extended discussion of Aristotelean syllogistic is the desire to promote contemporary logic amongst philosophers who were in the audience of his lecture. After all, in 1939 logic was everything but a fully established discipline, especially, in philosophy.

From a contemporary perspective a further very striking feature of Gödel's notes and the one aspect that, from this modern perspective, calls for thorough revision is Gödel's very sloppy approach to the use/mention distinction. While it is understandable to dispense of quotation marks in one's private notes, Gödel also equates objects and their names, predicates and properties, and, more generally, does not provide any precise definition of the formulas of the respective languages (save a definition of the language itself). Similarly, his discussion of the notion of strict implication, which he contrasts to material implication, is unclear and could be reconstructed as "committing the sin of confusing use and mention".[2] But no-one would accuse Gödel of being confused about use and mention, and, consequently, Gödel's sloppiness may show how little the awareness of the distinction is yet reflected in the logician's *lingua* at that time. Or, perhaps, Gödel just did not take the use/mention distinction to be particularly important and in need of explanation.

However, with the exception of his (non-) discussion of the use/mention distinction, Gödel's Basic Logic Course at Notre Dame is, as mentioned, surprisingly standard, even from the contemporary perspective. Gödel launches his investigation by discussing different truth-functional connectives and introduces truth tables. He spends some time motivating the truth table for material implication and discusses the so-called paradoxes of material implication. Truth tables are used to show the decidability of propositional logic, that is, we can decide of any given formula whether it is logically true or not. Gödel also discusses the functional completeness of propositional logic by illustrating for the case of three propositional variables how all resulting truth-functions can be defined using the truth functions for $\vee$ and $\neg$. However, we cannot use, as Gödel points out, every two-place connective together with negation. For example, if we choose the biconditional, or any other truth-function whose outputs consist of an even number of value T, no truth-function whose outputs consist of an uneven number, e.g. $\vee$ or $\wedge$, can be defined.

Having settled the question of primitives for his system Gödel introduces an axiomatic calculus for propositional logic following the outlines of Hilbert and Ackermann (1928). The calculus has four axioms and four rules of inference. The latter are the rule of uniform substitution, modus pones and what Gödel calls the *rule of defined symbol.* The rule of defined symbol allows for substitution of formulas with defined symbols for their definiens and *vice versa.* Gödel shows the calculus to be sound and complete. In the completeness proof Gödel appeals to the fact every formula is decided by a "fundamental conjunction" (p. 51) of its propo-

[2]Compare the discussion in (Marcus, 1961, p. 303) and Quine (1961) for this formulation. On p. 18 Gödel says: "$p \supset q$ could be given the meaning: q is a logical consequence of p, i.e. q can be derived from p by means of a chain syllogisms." On a charitable reconstruction this can be reconstructed as '$p \supset q$' is true iff 'q' is a logical consequence of 'p'. On this reconstruction there is no confusion about use and mention. But just before the above quote Gödel says: "We must not forget that $p \supset q$ was understood to mean simply 'if p then q' and nothing else (...)." This suggests that if $\supset$ is taken to stand for strict implication it should be read in terms of logical consequence, which conflates use and mention.

sitional variables, where a fundamental conjunction is a conjunction of propositional variables and negations thereof. But for a tautology one can show that it is implied by each of its fundamental conjunctions. Based on this observation it is then straightforward to show that every tautology can be proved. By establishing the completeness of his calculus Gödel has filled both gaps of traditional logic. He has provided a complete theory of logical inference and logically true propositions and has shown that all logically true propositions can be deduced from a minimum number of logical laws. The four axioms of his calculus form indeed a minimum number of logical laws because they are independent, as Gödel highlights. Gödel ends his discussion of propositional logic by introducing, inspired by the work of Gentzen, a sequent natural deduction system, which uses $\neg$ and $\supset$ as primitive symbols.

The discussion of predicate logic is more basic in comparison to the discussion of propositional logic, as important metatheoretical results like completeness, undecidability, and decidability of the monadic fragment of predicate logic are only mentioned rather than proved. Gödel introduces predicates and individuals, atomic propositions of subject-predicate form, quantifiers and the distinction between a bound and a free variable. He also makes some brief remarks on the notion of tautology and argues that "*an expression is a tautology in a world with infinitely many individuals*" (p. 76), that is, in more modern terms, a tautology has to be true independently of the cardinality of the domain. The axiom system Gödel gives is fairly standard and extends the propositional calculus by the axiom of universal instantiation and the rule of generalization in the consequence (together with some rather convoluted substitution rule).

The final theme Gödel explores in his lecture is the calculus of classes, Russell's paradox and the theory of types. Gödel introduces classes, i.e. sets, via predicate-abstracts, which for a given monadic predicate are supposed to stand for the extension of the predicate. He introduces a kind of naive comprehension principle and Extensionality and defines basic set-theoretic operations such as union, intersection, and complement. The lecture notes end with a discussion of the theory of types and Russell's paradox or, more generally, the paradoxes of the calculus of classes. The mechanisms that lead to paradox are illustrated by presenting a variant of Grelling's paradox. Interestingly, Gödel's diagnosis is (i) that self-reference is not to be blamed for paradox but rather (ii) that paradoxes arises since we falsely assume to be quantifying over the totality of objects whilst *"there does not exist a concept of the totality of all objects"* (p. 114). As a consequence, Gödel argues, we must quantify only over objects of a given type, that is, we should resort to a hierarchy of types to avoid the paradoxes—although the hierarchy is meant to be less restrictive than Russell's which attempts to block any form of self-reference. Indeed, Gödel's solution to Russell's paradox is somewhat similar in spirit to the response of contextualists such as Parsons (1974) and Glanzberg (2004) to the Liar paradox. But in the last lines of his notes Gödel argues that this strategy is not the correct solution to the *"so-called epistemic paradoxes"* (p. 115) such as Epimenides' paradox and, arguably, the Liar paradox. Unfortunately, at this point the notes end, and Gödel's thoughts, at this stage of his life, about how these paradoxes should be resolved remain a mystery. Despite this disappointing end Gödel's lecture notes of his Basic Logic Course at Notre Dame are a highly recommended read for logicians and philosophers who want to gain an insight into Gödel's thoughts at the time of the lecture and, more generally, the development of logic in the first half of the 20th century. Moreover, the insight comes without a cost since a free preprint of the book is available online.

Acknowledgments Johannes Stern's work has been supported by European Commission via the Marie Sklodowska-Curie Individual Fellowship TREPISTEME (Grant 703539).

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